Wiedemann-Franz Law – Solution

Part A: Electrical conductivity of metals (1.5 points)

A.1 (1.0 points)

Magnet descend time:

Number	Copper [s]	Aluminum[s]	Brass [s]
1	17.77	9.23	6.1
2	17.96	9.39	5.83
3	18.16	9.22	6.04
4	18.15	9.37	5.86
5	17.76	9.36	6.16
6	18.2	9.44	5.92
7	17.67	9.65	5.9
8	17.9	9.18	6.08
9	17.67	9.41	5.86
10	18.36	8.96	5.99
Average	17.96	9.32	5.97

A.2 (0.5 points)

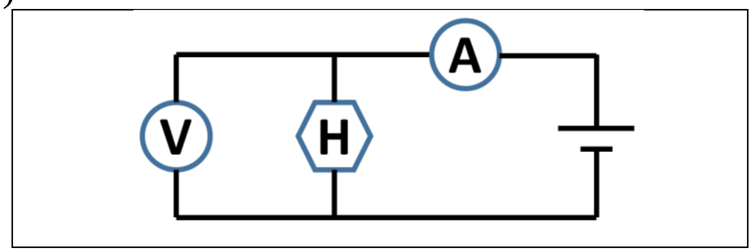
	Copper	Aluminum	Brass
Electrical conductivity $\left[\frac{1}{\Omega m}\right]$	5.97×10^7	2.98×10^7	1.60×10^7

Part B: Thermal conductivity of copper (3.0 points)

B.1 (0.1 points)

Rod 1 temperature : **22.76 [C]**

B.2 (0.5 points)



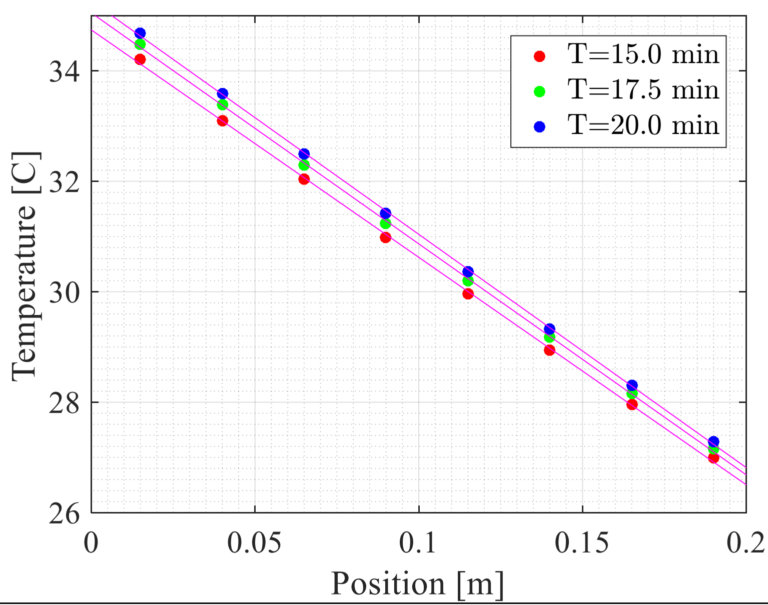
B.3 (0.1 points)

$P = I \cdot V = 5.51[\mathrm{W}]$

B.4 (0.5 points)

Time [S]	T1 [C]	T2 [C]	T3 [C]	T4[C]	T5 [C]	T6 [C]	T7 [C]	T8 [C]
900	26.98	27.96	28.95	29.96	30.98	32.03	33.10	34.20
1050	27.16	28.16	29.17	30.20	31.240	32.30	33.38	34.48
1200	27.29	28.30	29.33	30.37	31.42	32.49	33.58	34.68

B.5 (1.0 points)



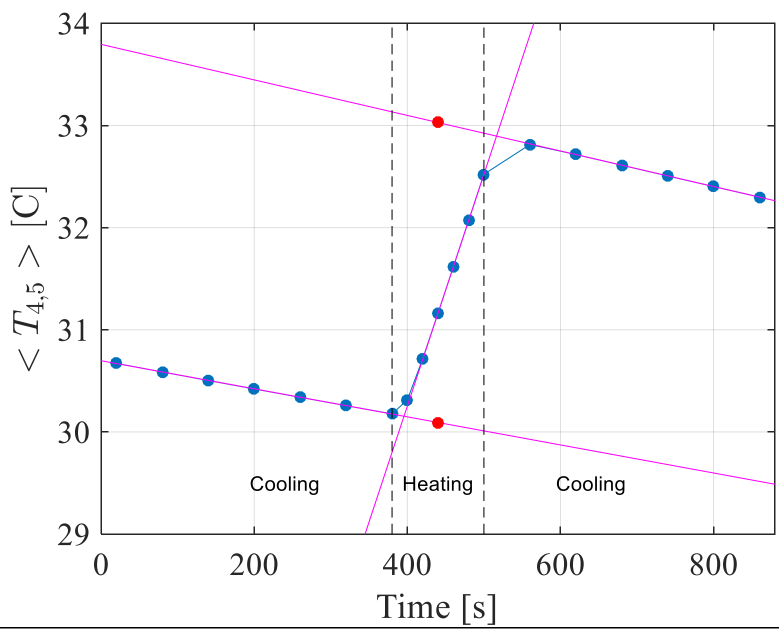
B.6 (0.5 points) $\kappa_0 = -\frac{P}{A\frac{\Delta T}{\Delta x}} = -\frac{5.51[W]}{\pi\cdot\left(10^{-2}[m]\right)^2\cdot\left(-41.8\left[\frac{K}{m}\right]\right)} = 420\left[\frac{W}{mK}\right]$ $\frac{\Delta T}{\Delta t} = \frac{31.04[C]-30.62[C]}{5\cdot 60[s]} = 1.4\cdot 10^{-3}\left[\frac{K}{s}\right]$
B.7 (0.3 points) **<u>higher value</u>** We expect a **<u>higher value</u>** of κ_0 compared with the real κ_{cu} because of 2 reasons: 1. A part of the supplied heat power is lost through the side walls. Therefore, the heat transfer through the cross-section of the rod is smaller. 2. Since the system is not in a steady state ($\frac{\Delta T}{\Delta t} \neq 0$), the corresponding power involved should be subtracted from the power supplied by the heater.

Part C: Heat loss and heat capacity of copper (4.0 points)

C.1 (1.0 points)

$Time[s]$	$T_1[C]$	$T_2[C]$	$T_3[C]$	$T_4[C]$	$T_5[C]$	$T_6[C]$	$T_7[C]$	$T_8[C]$	$T_{av}[C]$
20				30.67	30.67				30.67
80				30.59	30.59				30.59
140				30.50	30.50				30.50
200				30.42	30.42				30.42
260				30.34	30.34				30.34
320				30.26	30.26				30.26
380				30.18	30.18				30.18
400				30.38	30.25				30.31
420				30.87	30.56				30.72
440				31.37	30.96				31.16
460				31.85	31.38				31.61
480				32.32	31.82				32.07
500				32.78	32.26				32.52
560				32.88	32.75				32.81
620				32.73	32.70				32.72
680				32.61	32.61				32.61
740				32.51	32.51				32.51
800				32.40	32.40				32.40
860				32.30	32.30				32.30

C.2 (1.0 points)



C.3 (1.0 points)

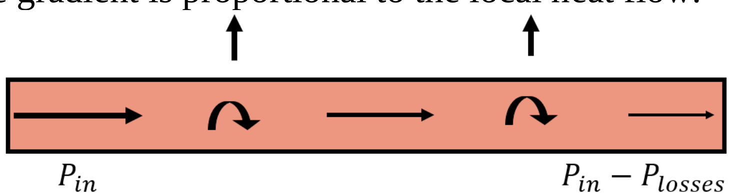
The purpose of this part is to correct to first order the result in part B. Hence, every solution within 10% accuracy is accepted (see marking scheme).

Solution 1 (using slopes):	Solution 2 (using jump):
$P_{loss} = c_p \cdot m \cdot \left.\frac{\partial T_{av}}{\partial t}\right\|_{Cooling}$ $P_{in} = c_p \cdot m \cdot \left(\left.\frac{\partial T_{av}}{\partial t}\right\|_{Heating} - \left.\frac{\partial T_{av}}{\partial t}\right\|_{Cooling} \right)$ Where $\left.\frac{\partial T_{av}}{\partial t}\right\|_{Cooling}$ is the average of both cooling slopes. $c_p \cdot m = \frac{5.5[W]}{\left(2.27\cdot 10^{-2}\left[\frac{K}{s}\right] + 1.6\cdot 10^{-3}\left[\frac{K}{s}\right]\right)}$	$P_{loss} = c_p \cdot m \cdot \left.\frac{\partial T_{av}}{\partial t}\right\|_{Cooling}$ $P_{in} \cdot \Delta t = c_p \cdot m \cdot \Delta T$ Where $\left.\frac{\partial T_{av}}{\partial t}\right\|_{Cooling}$ is the average of the two cooling slopes, and ΔT is the extrapolated jump in temperature half way though the heating time interval. $c_p \cdot m = \frac{P_{in} \cdot \Delta t}{\Delta T} = \frac{5.5[W]\cdot 120[s]}{2.94[K]} = 224\left[\frac{J}{K}\right]$

$c_p \cdot m = 226\left[\frac{J}{K}\right] \Rightarrow c_p = 390\left[\frac{J}{kg\cdot K}\right]$ Which is 1% off the correct value. $P_{loss} = 226\left[\frac{J}{K}\right]\cdot 1.4\cdot 10^{-3}\left[\frac{K}{s}\right] = 0.32[W]$	$c_p = 386\left[\frac{J}{kg\cdot K}\right]$ which is the correct value. $P_{loss} = 224\left[\frac{J}{K}\right]\cdot 1.4\cdot 10^{-3}\left[\frac{K}{s}\right] = 0.31[W]$

C.4 (1.0 points)

The temperature gradient is proportional to the local heat flow.



To first order, the average temperature gradient will be proportional to the average heat flow. Therefore, the temperature gradient will be proportional to $P_{in} - \frac{1}{2}P_{losses}$:

$$\kappa = \frac{P_{in} - \frac{1}{2}P_{absorb} - \frac{1}{2}P_{loss}}{A\cdot(\Delta T/\Delta x)} = \frac{P_{in} - \frac{1}{2}c_p\cdot m\cdot\frac{\Delta T}{\Delta t} - \frac{1}{2}\dot{Q}_{loss}}{A\cdot\Delta T/\Delta x} = \kappa_0\cdot\frac{P_{in} - \frac{1}{2}c_p\cdot m\cdot\frac{\Delta T}{\Delta t} - \frac{1}{2}\dot{Q}_{loss}}{P}$$

$$\kappa = 420\left[\frac{W}{mK}\right]\cdot\frac{5.51[W] - \frac{1}{2}\cdot 226\left[\frac{J}{K}\right]\cdot 1.4\cdot 10^{-3}\left[\frac{K}{s}\right] - \frac{1}{2}\cdot 0.32[W]}{5.51[W]} = 396\left[\frac{W}{mK}\right]$$

Which gives an error of 2.5% error compared to expected $385\left[\frac{W}{mK}\right]$. We expect a 1% systematic error (see appendix).

Part D: Thermal conductivity of multiple metals (1.0 points)

D.1 (0.1 points)

$T = 22.65[C]$

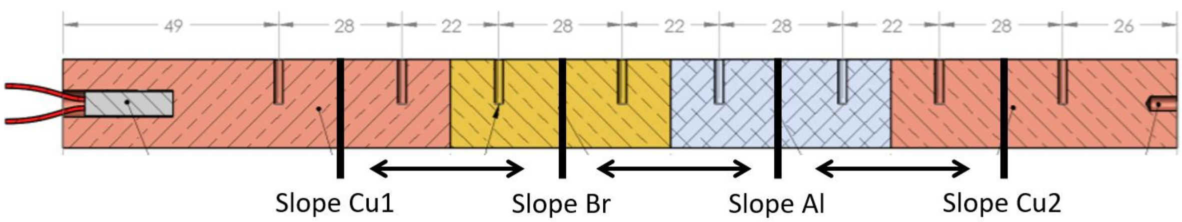
D.2 (0.2 points)

Time of measurement: $1041[s]$

$T_1[C]$	$T_2[C]$	$T_3[C]$	$T_4[C]$	$T_5[C]$	$T_6[C]$	$T_7[C]$	$T_8[C]$
41.68	40.51	38.51	34.65	32.47	30.71	29.63	28.62

$\Delta T_{cu1}/\Delta x$	$\Delta T_{Br}/\Delta x$	$\Delta T_{Al}/\Delta x$	$\Delta T_{cu2}/\Delta x$
$41.79\left[\frac{K}{m}\right]$	$137.86\left[\frac{K}{m}\right]$	$62.86\left[\frac{K}{m}\right]$	$36.07\left[\frac{K}{m}\right]$

D.3 (0.7 points)



$$\kappa_{Brass} = \kappa_{Copper} \cdot \frac{\frac{2}{3}(\Delta T_{cu1}/\Delta x)+\frac{1}{3}(\Delta T_{cu2}/\Delta x)}{\Delta T_{Br}/\Delta x} = 115\left[\frac{W}{mK}\right]$$

$$\kappa_{Aluminum} = \kappa_{Copper} \cdot \frac{\frac{1}{3}(\Delta T_{cu1}/\Delta x)+\frac{2}{3}(\Delta T_{cu2}/\Delta x)}{\Delta T_{Al}/\Delta x} = 239\left[\frac{W}{m\cdot K}\right]$$

Part E: The Wiedemann-Franz law (0.5 points)

E.1 (0.5 points)

	Copper	Aluminum	Brass
$\sigma\ [\Omega^{-1}m^{-1}]$ Electric conductivity	5.97×10^7	2.98×10^7	1.60×10^7
$\kappa \left[\frac{W}{Km}\right]$ Heat conductivity	396	239	115
$L \left[\frac{W\Omega}{K^2}\right]$ Lorenz coefficient	2.21×10^{-8}	2.67×10^{-8}	2.40×10^{-8}